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SU(3) analysis of annihilation contributions and CP violating relations in $B \to PP$ decays

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Abstract. Several methods proposed to measure the angle γ in the KM unitarity triangle assumed that the tree contribution to $B^- \to \pi^- \bar{K}^0$ is purely due to annihilation contributions and is negligibly small. This assumption has to be tested in order to have a correct interpretation of the experimental data. In this paper we show that using SU(3) symmetry, the smallness of the tree contribution can be tested in a dynamic model-independent way. We also derive several relations between CP violating rate differences for $B \to PP$ decays without assuming the smallness of the annihilation contributions. These relations provide important tests for the standard model of CP violation.

1 Introduction

Several rare two body decay modes of $B_{u,d}$ mesons have been observed at CLEO [1]. These data have provided interesting information about the standard model (SM) [2–5]. With increased luminosities for the B factories at CLEO, KEK and SLAC, more useful information about rare $B_{u,d}$ decays will be obtained. The SM will be tested in detail. At present the study of rare B_s decays is limited by statistics. Only some weak upper limits on the branching ratios have been obtained [6]. However, more data on B_s decays will become available from LHC in the future. These data will help to further test the SM [2,7]. Theoretical predictions are, however, limited by our inability to reliably calculate many hadronic matrix elements related to B decays. This prevents a full test of the standard model. Lacking reliable calculations, attempts have been made to extract useful information from symmetry considerations. SU(3) flavor symmetry is one of the symmetries which has attracted a lot of attention recently [8–10]. For example, it has been shown that using SU(3) symmetry it is possible to constrain [11] and to determine [4,12] one of the fundamental parameters, namely γ, in the SM for CP violation by measuring several B meson decay modes.

Some of the methods proposed to measure γ depend on the assumption that the tree amplitude for $B^- \to \pi^- \bar{K}^0$ is negligibly small [10,12]. To correctly interpret the experimental data, the smallness of the tree contribution has to be confirmed experimentally. It is often assumed that the tree amplitude for $B^- \to \pi^- \bar{K}^0$ receives annihilation contributions only. If this is true, one has to make sure that these contributions are small. Of course one has to make sure that it is true that the decay amplitude is dominated by annihilation contributions. There have been several discussions of constraining the annihilation contributions using an SU(3) analysis [13]. In this paper we will use SU(3) symmetry to study further related problems, but look at the problems in a different angle. We will first show how one can use SU(3) relations to test the smallness of annihilation contributions. We then show that the statement that the tree amplitude receives annihilation contributions only for $B^- \to \pi^- \bar{K}^0$ is not strictly a SU(3) result. We will show how to verify the smallness of the tree amplitude for $B^- \to \pi^- \bar{K}^0$ using several B decay modes. Finally, we will use SU(3) symmetry to derive several useful relations regarding CP violating rate differences without any assumption about the size of the annihilation contributions. These relations provide further tests for the SM of CP violation and also the SU(3) symmetry.

2 SU(3) decay amplitudes for $B \to PP$

The quark-level effective Hamiltonian up to one-loop level in the electroweak interaction for hadronic charmless B decays, including the corrections to the matrix elements, can be written as

$$H^q_{\rm eff} = \frac{4G_F}{\sqrt{2}}\Big[V_{ub}V^*_{uq}(c_1O_1 + c_2O_2) - \sum_{i=3}^{12}(V_{ub}V^*_{uq}c_i^{uc} + V_{tb}V^*_{tq}c_i^{tc})O_i\Big]. \tag{1}$$

The operators are defined in [14]. The coefficients $c_{1,2}$ and $c_i^{jk} = c_i^j - c_i^k$, with j indicating the internal quark, are the Wilson coefficients (WC). These WC's have been evaluated by several groups [14], with $|c_{1,2}| >> |c_i^j|$. In the above the factor $V_{cb}V^*_{cq}$ has been eliminated using the unitarity property of the KM matrix.

At the hadronic level, the decay amplitude can be generically written as

$$A = \langle \text{final state}|H^q_{\text{eff}}|B\rangle = V_{ub}V^*_{uq}T(q) + V_{tb}V^*_{tq}P(q), \quad (2)$$

where $T(q)$ contains contributions of the *tree* as well as *penguin* type due to charm and up quark loop corrections to the matrix elements, while $P(q)$ contains contributions purely from *penguin* due to top and charm loops. We would like to clarify the notation used here. The amplitude T in (2) is usually called the "tree" amplitude; it will also be referred to later on in the paper. One should, however, keep in mind that it really contains the usual tree current–current contributions proportional to $c_{1,2}$ and also the u and c penguin contributions proportional to $c^u_i - c^c_i$ with $i = 3$–10 which is small due to cancellation between u and c internal quark contributions. Also, in general, it contains long-distance contributions corresponding to internal u and c generated intermediate hadron states [15]. In the cases where the tree current–current contributions are zero or very small, like the case of $B^- \to \pi^- \bar{K}^0$ from a factorization calculation, the penguin contributions may play an important role. In our later analysis, we do not distinguish the tree and the penguin contributions in the amplitude T and try to find ways to test the smallness in certain decays.

The relative strength of the amplitudes T and P is predominantly determined by their corresponding WC's in the effective Hamiltonian. For $\Delta S = 0$ charmless decays, the dominant contributions are due to the tree operators $O_{1,2}$ and the penguin operators are suppressed by smaller WC's, whereas for $\Delta S = -1$ decays, because the penguin contributions are enhanced by a factor of $V_{tb}V^*_{ts}/V_{ub}V^*_{us} \approx 55$ compared with the tree contributions, penguin effects dominate the decay amplitudes. In this case the electroweak penguins can also play a very important role [16], in particular when studying CP violation in B decays [17]. One should carefully keep track of the different contributions.

The operators $O_{1,2}$, $O_{3\text{–}6,11,12}$, and $O_{7\text{–}10}$ transform under SU(3) as $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, $\bar{3}$, and $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, respectively. These properties enable us to write the decay amplitudes for $B \to PP$ in only a few SU(3) invariant amplitudes.

For the $T(q)$ amplitude, for example, we have [9]

$$\begin{aligned} T(q) = {} & A^{\rm T}_{\bar{3}} B_i H(\bar{3})^i (M^k_l M^l_k) + C^{\rm T}_{\bar{3}} B_i M^i_k M^k_j H(\bar{3})^j \\ & + A^{\rm T}_6 B_i H(6)^{ij}_k M^l_j M^k_l + C^{\rm T}_6 B_i M^i_j H(6)^{jk}_l M^l_k \\ & + A^{\rm T}_{\overline{15}} B_i H(\overline{15})^{ij}_k M^l_j M^k_l + C^{\rm T}_{\overline{15}} B_i M^i_j H(\overline{15})^{jk}_l M^l_k, \end{aligned} \quad (3)$$

where $B_i = (B_u, B_d, B_s) = (B^-, \bar{B}^0, \bar{B}^0_s)$ is a SU(3) triplet, M^j_i is the SU(3) pseudoscalar octet, and the matrices $H(i)$ contain information on the transformation properties of the operators $O_{1\text{–}12}$.

For $q = d$, the non-zero entries of the matrices $H(i)$ are given by

$$\begin{aligned} & H(\bar{3})^2 = 1,\ \ H(6)^{12}_1 = H(6)^{23}_3 = 1, \\ & H(6)^{21}_1 = H(6)^{32}_3 = -1,\ \ H(\overline{15})^{12}_1 = H(\overline{15})^{21}_1 = 3, \\ & H(\overline{15})^{22}_2 = -2,\ H(\overline{15})^{32}_3 = H(\overline{15})^{23}_3 = -1, \end{aligned} \quad (4)$$

and for $q = s$, the non-zero entries are

$$\begin{aligned} & H(\bar{3})^3 = 1,\ \ H(6)^{13}_1 = H(6)^{32}_2 = 1, \\ & H(6)^{31}_1 = H(6)^{23}_2 = -1,\ \ H(\overline{15})^{13}_1 = H(\overline{15})^{31}_1 = 3, \\ & H(\overline{15})^{33}_3 = -2,\ \ H(\overline{15})^{32}_2 = H(\overline{15})^{23}_2 = -1. \end{aligned} \quad (5)$$

Due to the anti-symmetry property of $H(6)$ in exchanging the upper two indices, A_6 and C_6 are not independent [9]. For individual decay amplitudes, A_6 and C_6 always appear together in the form $C_6 - A_6$. We will absorb A_6 in the definition of C_6. In terms of the SU(3) invariant amplitudes, the decay amplitudes for the various B meson decays are given by

$\Delta S = 0$

$$\begin{aligned}
T^{B_u}_{\pi^-\pi^0}(d) &= \frac{8}{\sqrt{2}} C^{\rm T}_{\overline{15}}, \\
T^{B_u}_{\pi^-\eta_8}(d) &= \frac{2}{\sqrt{6}} (C^{\rm T}_{\bar{3}} - C^{\rm T}_6 + 3A^{\rm T}_{\overline{15}} + 3C_{\overline{15}}), \\
T^{B_u}_{K^-K^0}(d) &= C^{\rm T}_{\bar{3}} - C^{\rm T}_6 + 3A^{\rm T}_{\overline{15}} - C^{\rm T}_{\overline{15}}, \\
T^{B_d}_{\pi^+\pi^-}(d) &= 2A^{\rm T}_{\bar{3}} + C^{\rm T}_{\bar{3}} + C^{\rm T}_6 + A^{\rm T}_{\overline{15}} + 3C^{\rm T}_{\overline{15}}, \\
T^{B_d}_{\pi^0\pi^0}(d) &= \frac{1}{\sqrt{2}} (2A^{\rm T}_{\bar{3}} + C^{\rm T}_{\bar{3}} + C^{\rm T}_6 + A^{\rm T}_{\overline{15}} - 5C^{\rm T}_{\overline{15}}), \\
T^{B_d}_{K^-K^+}(d) &= 2(A^{\rm T}_{\bar{3}} + A^{\rm T}_{\overline{15}}), \\
T^{B_d}_{\bar{K}^0K^0}(d) &= 2A_{\bar{3}} + C^{\rm T}_{\bar{3}} - C^{\rm T}_6 - 3A^{\rm T}_{\overline{15}} - C_{\overline{15}}, \\
T^{B_d}_{\pi^0\eta_8}(d) &= \frac{1}{\sqrt{3}} (-C^{\rm T}_{\bar{3}} + C^{\rm T}_6 + 5A^{\rm T}_{\overline{15}} + C_{\overline{15}}), \\
T^{B_d}_{\eta_8\eta_8}(d) &= \frac{1}{\sqrt{2}} \left(2A_{\bar{3}} + \frac{1}{3} C^{\rm T}_{\bar{3}} - C^{\rm T}_6 - A^{\rm T}_{\overline{15}} + C_{\overline{15}} \right), \\
T^{B_s}_{K^+\pi^-}(d) &= C^{\rm T}_{\bar{3}} + C^{\rm T}_6 - A^{\rm T}_{\overline{15}} + 3C_{\overline{15}}, \\
T^{B_s}_{K^0\pi^0}(d) &= -\frac{1}{\sqrt{2}} (C^{\rm T}_{\bar{3}} + C^{\rm T}_6 - A^{\rm T}_{\overline{15}} - 5C_{\overline{15}}), \\
T^{B_s}_{K^0\eta_8}(d) &= -\frac{1}{\sqrt{6}} (C^{\rm T}_{\bar{3}} + C^{\rm T}_6 - A^{\rm T}_{\overline{15}} - 5C_{\overline{15}}),
\end{aligned}$$

$\Delta S = -1$

$$\begin{aligned}
T^{B_u}_{\pi^-\bar{K}^0}(s) &= C^{\rm T}_{\bar{3}} - C^{\rm T}_6 + 3A^{\rm T}_{\overline{15}} - C^{\rm T}_{\overline{15}}, \\
T^{B_u}_{\pi^0K^-}(s) &= \frac{1}{\sqrt{2}} (C^{\rm T}_{\bar{3}} - C^{\rm T}_6 + 3A^{\rm T}_{\overline{15}} + 7C^{\rm T}_{\overline{15}}), \\
T^{B_u}_{\eta_8K^-}(s) &= \frac{1}{\sqrt{6}} (-C^{\rm T}_{\bar{3}} + C^{\rm T}_6 - 3A^{\rm T}_{\overline{15}} + 9C^{\rm T}_{\overline{15}}), \\
T^{B_d}_{\pi^+K^-}(s) &= C^{\rm T}_{\bar{3}} + C^{\rm T}_6 - A^{\rm T}_{\overline{15}} + 3C^{\rm T}_{\overline{15}}, \\
T^{B_d}_{\pi^0\bar{K}^0}(s) &= -\frac{1}{\sqrt{2}} (C^{\rm T}_{\bar{3}} + C^{\rm T}_6 - A^{\rm T}_{\overline{15}} - 5C^{\rm T}_{\overline{15}}), \\
T^{B_d}_{\eta_8\bar{K}^0}(s) &= -\frac{1}{\sqrt{6}} (C^{\rm T}_{\bar{3}} + C^{\rm T}_6 - A^{\rm T}_{\overline{15}} - 5C^{\rm T}_{\overline{15}}), \\
T^{B_s}_{\pi^+\pi^-}(s) &= 2(A^{\rm T}_{\bar{3}} + A^{\rm T}_{\overline{15}}), \\
T^{B_s}_{\pi^0\pi^0}(s) &= \sqrt{2}(A^{\rm T}_{\bar{3}} + A^{\rm T}_{\overline{15}}), \\
T^{B_s}_{K^+K^-}(s) &= 2A^{\rm T}_{\bar{3}} + C^{\rm T}_{\bar{3}} + C^{\rm T}_6 + A^{\rm T}_{\overline{15}} + 3C^{\rm T}_{\overline{15}}, \\
T^{B_s}_{K^0\bar{K}^0}(s) &= 2A^{\rm T}_{\bar{3}} + C^{\rm T}_{\bar{3}} - C^{\rm T}_6 - 3A^{\rm T}_{\overline{15}} - C^{\rm T}_{\overline{15}},
\end{aligned}$$

$$T^{B_s}_{\pi^0\eta_8}(s) = \frac{2}{\sqrt{3}}(C^{\mathrm{T}}_6 + 2A^{\mathrm{T}}_{\overline{15}} - 2C^{\mathrm{T}}_{\overline{15}}),$$
$$T^{B_s}_{\eta_8\eta_8}(s) = \sqrt{2}\left(A^{\mathrm{T}}_{\bar{3}} + \frac{2}{3}C^{\mathrm{T}}_{\bar{3}} - A^{\mathrm{T}}_{\overline{15}} - 2C^{\mathrm{T}}_{\overline{15}}\right).$$

The amplitudes for $P(q)$ in terms of the SU(3) invariant amplitudes can be obtained in a similar way. We will denote the corresponding amplitudes by A^P_i and C^P_i.

Many analyses have been carried out using an SU(3) classification of the quark-level diagrams [10]. In most cases such an analysis will yield the same results as the use of SU(3) invariant amplitudes. However, in some cases a careless classification according to quark-level diagrams would mean the loss of some vital information. An interesting example is the tree amplitude for $B^- \to \pi^- \bar{K}^0$. Using quark-level diagram analysis, when the annihilation contributions are neglected, the current–current operators do not contribute to this decay. This implies, if the small contributions from u and c penguins are neglected, in the SU(3) invariant amplitude language, that

$$C^{\mathrm{T}}_{\bar{3}} - C^{\mathrm{T}}_6 - C^{\mathrm{T}}_{\overline{15}} = 0. \quad (6)$$

This, however, is not generally true as has been confirmed by model calculations [18,19]. In the quark-level diagram classification there are only four independent amplitudes, whereas in the general SU(3) invariant classification there are five independent amplitudes [9]. Some information related to different combinations of quark-level diagrams and their phases have been lost in the naive quark-level diagram analysis. Specifically, four quark operators containing $\bar{d}\Gamma_1 d\bar{q}\Gamma_2 b$ and $\bar{s}\Gamma_1 s\bar{q}\Gamma_2 b$ types of terms, where Γ_i denote the appropriate Dirac matrices, and appearing in the SU(3) invariant amplitudes, do not appear in the naive tree quark diagram analysis. For this reason, we will use the SU(3) invariant amplitude to carry out our analysis.

3 Test of the smallness of annihilation contributions

The amplitudes $A_{\bar{3},\overline{15}}$ correspond to annihilation contributions. Here, we take the amplitudes with one of the light quarks in the effective Hamiltonian corresponding to the light quark inside the B mesons to be annihilation amplitudes. The amplitudes $A_{\bar{3},\overline{15}}$ are annihilation amplitudes as can be understood by noticing that the light-quark index in the B mesons are contracted with the Hamiltonian [20]. The A and E type of contributions in the quark diagram classification are linear combinations of $A_{\bar{3}}$ and $A_{\overline{15}}$. Based on model calculations [10] it has been argued that these contributions are small. At present the annihilation contributions cannot reliably be calculated. In view of this, it is important to be able to test the smallness of the annihilation contributions experimentally.

In this section we show that using SU(3) relations, the size of the annihilation contributions can be measured independent of dynamic models for the matrix elements, and therefore the smallness of these amplitudes can be tested. Two types of tests can be carried out. One of them is to test the smallness of the annihilation contributions of the SU(3) invariant amplitudes, and the other is to test the smallness of the tree contribution to $B^- \to \pi^- \bar{K}^0$.

The best way to test the smallness of the annihilation contributions is to use processes involving only $A_{\bar{3},\overline{15}}$. From the discussions of the previous section, we find that there are only three such processes. They are: (a) $\bar{B}^0 \to K^+K^-$ [21]; (b) $B_s \to \pi^-\pi^+$; and (c) $B_s \to \pi^0\pi^0$. Their decay amplitudes are given by

$$\begin{aligned}
&A(B_d \to K^+K^-) = 2V_{ub}V^*_{ud}(A^{\mathrm{T}}_{\bar{3}} + A^{\mathrm{T}}_{\overline{15}}) \\
&+2V_{tb}V^*_{td}(A^P_{\bar{3}} + A^P_{\overline{15}}), \\
&A(B_s \to \pi^-\pi^+) = 2V_{ub}V^*_{us}(A^{\mathrm{T}}_{\bar{3}} + A^{\mathrm{T}}_{\overline{15}}) \\
&+2V_{tb}V^*_{ts}(A^P_{\bar{3}} + A^P_{\overline{15}}), \\
&A(B_s \to \pi^0\pi^0) = \frac{1}{\sqrt{2}}A(B_s \to \pi^+\pi^-). \quad (7)
\end{aligned}$$

It is clear that these decays receive annihilation contributions only. However, there is a crucial difference between (a), and (b) and (c). The decay amplitude for (a) is dominated by the tree contribution and the amplitudes for (b) and (c), being $\Delta S = -1$ processes, are dominated by penguin contributions. If annihilation contributions are small, these processes will all have small branching ratios. At present, these three modes have not been observed. The best constraint is from $\bar{B}^0 \to K^+K^-$ with an upper bound on the branching ratio of 0.24×10^{-5} at the 90% confidence level from CLEO [1]. However, this still allows substantial annihilation contributions. The annihilation contributions to the tree amplitude to $B^- \to \pi^-\bar{K}^0$ can reach 10% of the total amplitude. We have to wait for more data to verify the smallness of the annihilation contributions. Conclusions drawn with such an assumption should be viewed with caution.

One should be aware that even if the annihilation contributions are small, this does not mean that the tree amplitude for $B^- \to \pi^-\bar{K}^0$ is small. One also has to verify that the tree amplitude receives annihilation contributions only. Let us now study how this can be verified. From the SU(3) decay amplitudes listed in the previous section, we see that the tree contribution to this process is given by

$$T^{B_u}_{\pi^-\bar{K}^0}(s) = C^{\mathrm{T}}_{\bar{3}} - C^{\mathrm{T}}_6 + 3A^{\mathrm{T}}_{\overline{15}} - C^{\mathrm{T}}_{\overline{15}}. \quad (8)$$

This is not a pure annihilation process as for $\bar{B}^0 \to K^+K^-$, $B_s \to \pi^-\pi^+$ and $B_s \to \pi^0\pi^0$. The tree amplitude to $B^- \to \pi^-\bar{K}^0$ is a pure annihilation contribution only if $C^{\mathrm{T}}_{\bar{3}} - C^{\mathrm{T}}_6 - C^{\mathrm{T}}_{\overline{15}} = 0$. In order for this to be true, not only the magnitude of the invariant amplitudes should be arranged, but also the phases of these amplitudes must be arranged to have the cancellation. However, from our experience with K and D systems, we know that different SU(3) (or isospin) amplitudes develop different phases. It is quite possible that the same situation happens in B systems [22,23]. To have a better understanding of the situation, let us perform a calculation of the tree decay amplitude for $T(B^- \to \bar{K}^0\pi^-)$ in the factorization approximation neglecting the annihilation contributions, but with insertions of possible final-state interaction phases for different amplitudes. We have [3,17]

$$\begin{aligned}
&T(B^- \to \pi^- \bar{K}^0) = V_{ub}V_{us}^*(\mathrm{e}^{\mathrm{i}\delta_1}T_1 - \mathrm{e}^{\mathrm{i}\delta_3}T_3),\\
&T_1 = T_3 = \frac{1}{3}\mathrm{i}\frac{G_F}{\sqrt{2}}\left[\left(c_1 + \frac{c_2}{N}\right) f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2)\right.\\
&\left. + \left(\frac{c_1}{N} + c_2\right) f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)\right], \qquad (9)
\end{aligned}$$

where N is the number of colors. We have used the following definitions for the decay constants and form factors:

$$\begin{aligned}
&\langle P|\bar{q}\gamma_\mu(1-\gamma_5)u|0\rangle = \mathrm{i}f_P P_\mu,\\
&\langle P(k)|\bar{q}\gamma_\mu b|\bar{B}^0(p)\rangle = (k+p)_\mu F_1^{BP}\\
&+(m_P^2 - m_B^2)\frac{q_\mu}{q^2}(F_1^{BP}(q^2) - F_0^{BP}(q^2)), \qquad (10)
\end{aligned}$$

where $q = p - k$. The first term $\mathrm{e}^{\mathrm{i}\delta_1}T_1$ in the amplitude $T(B^- \to \pi^- \bar{K}^0)$ is equal to $C_{\bar{3}}^{\mathrm{T}} - C_6^{\mathrm{T}}$ which is an $I = 1/2$ amplitude, while the second term $\mathrm{e}^{\mathrm{i}\delta_3}T_3$ is equal to $C_{\overline{15}}^{\mathrm{T}}$, which is an $I = 3/2$ amplitude. We see that the cancellation happens only when $\delta_1 = \delta_3$, which is an additional assumption about the dynamics beyond SU(3) symmetry. It has been shown that the present data do not exclude a large final phase difference $\delta_1 - \delta_3$ [3,23]. The smallness of $C_{\bar{3}} - C_6 - C_{\overline{15}}$ has to be tested experimentally.

To have a model-independent test of this cancellation, that is, $C_{\bar{3}} - C_6 - C_{\overline{15}} = 0$, one needs to find processes which depend on the same combination of the SU(3) invariant amplitudes as the tree amplitude for $B^- \to \pi^- \bar{K}^0$. To this end, we carry out an analysis similar to [19] for $B^- \to K^- K^0$ using the parametrization of the SU(3) decay amplitudes in the previous section. Using $B^- \to K^- K^0$ decay to obtain useful information for the different amplitudes is also studied in [15]. We have

$$A(B^-\to K^-K^0) = V_{ub}V_{ud}^* T^{B_u}_{K^-K^0}(d) + V_{tb}V_{td}^* P^{B_u}_{K^-K^0}(d). \qquad (11)$$

As has been mentioned earlier, the relative strength of the T and P amplitudes is predominantly determined by their WC's, and to a good approximation $A(B^- \to K^-K^0) \approx V_{ub}V_{ud}^* T^{B_u}_{K^-K^0}(d)$. In the SU(3) limit

$$T^{B_u}_{\pi^-\bar{K}^0}(s) = T^{B_u}_{K^-K^0}(d) = C_{\bar{3}} - C_6 + 3A_{\overline{15}} - C_{\overline{15}}. \qquad (12)$$

Once the branching ratio for $B^- \to K^-K^0$ is measured, we have information about the size of $|T^{B_u}_{\bar{K}^0\pi^-}|$. If experimentally the branching ratio $B^- \to K^-K^0$ indeed turns out to be small, this would confirm the smallness of $C_{\bar{3}} - C_6 - C_{\overline{15}}$ if annihilation contributions are also found to be small from the branching ratio measurements for $\bar{B}^0 \to K^+K^-$, $B_s \to \pi^+\pi^-$ and $B_s \to \pi^0\pi^0$. In this case, conclusions drawn with the assumption $T^{B_u}_{\pi^-\bar{K}^0}(s) = 0$ would be correct. Otherwise, the results obtained with this assumption cannot be trusted. Unfortunately, at the present experimental upper bound, with $Br(B^- \to K^-K^0) < 0.93 \times 10^{-5}$ at the 90% confidence level from CLEO [1], still large tree contributions to $B^- \to \pi^- \bar{K}^0$ are allowed.

We stress that the smallness of the annihilation contributions and the smallness of the tree amplitude for $B^- \to \pi^- \bar{K}^0$ are two independent assumptions and should be tested separately as discussed in the above. These tests have important implications for the determination of the angle γ in the KM unitarity triangle, because some of the methods proposed require that the tree amplitude is small so that $A(B^- \to \pi^- \bar{K}^0) = \bar{A}(B^+ \to \pi^+ K^0)$. At present this is not well tested. We have to wait for future experiments to learn more.

4 CP asymmetry relation between B decays

From the previous discussions, we see that predictions with certain dynamic assumptions about the amplitudes suffer from possible uncertainties and need to be tested. It is desirable that tests for the SM can be performed in a dynamic model independent way. In this section we will derive several such relations, which can be used to test the standard model. These relations are related to the CP violation rate difference defined as

$$\Delta(B \to PP) = \Gamma(B \to PP) - \Gamma(\bar{B} \to \bar{P}\bar{P}). \qquad (13)$$

SU(3) symmetry relates $\Delta S = 0$ and $\Delta S = -1$ decays. One particularly interesting class of relations are the ones with $T(d) = T(s) = T$ and $P(d) = P(s) = P$. For this class of decays, we have [20,24]

$$\begin{aligned}
A(d) &= V_{ub}V_{ud}^* T + V_{tb}V_{td}^* P,\\
A(s) &= V_{ub}V_{us}^* T + V_{tb}V_{ts}^* P. \qquad (14)
\end{aligned}$$

Due to the different KM matrix elements involved in $A(d)$ and $A(s)$, although the amplitudes have some similarities, the branching ratios are not simply related. However, when considering the rate difference, $\Delta(B \to PP)$, the situation is dramatically different. Because of the simple property of the KM matrix element [25], $\mathrm{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -\mathrm{Im}(V_{ub}V_{us}^*V_{tb}^*V_{ts})$, we find that in the SU(3) limit

$$\Delta(d) = -\Delta(s), \qquad (15)$$

where $\Delta(i) = (|A(i)|^2 - |\bar{A}(i)|^2)\lambda_{ab}/(8\pi m_B)$ is the CP violating rate difference defined earlier and $\lambda_{ab} = (1 - 2(m_a^2 + m_b^2)/m_B^2 + (m_a^2 - m_b^2)^2/m_B^4)^{1/2}$ with $m_{a,b}$ being the masses of the two particles in the final state.

In the SU(3) limit we find the following equalities:

$$\begin{aligned}
&(1) \quad \Delta(B^- \to K^-K^0) = -\Delta(B^- \to \pi^-\bar{K}^0),\\
&(2) \quad \Delta(\bar{B}^0 \to \pi^-\pi^+) = -\Delta(B_s \to K^-K^+),\\
&(3) \quad \Delta(\bar{B}^0 \to K^-K^+) = -\Delta(B_s \to \pi^-\pi^+)\\
&\qquad = -2\Delta(B_s \to \pi^0\pi^0),\\
&(4) \quad \Delta(\bar{B}^0 \to \bar{K}^0K^0) = -\Delta(B_s \to K^0\bar{K}^0),\\
&(5) \quad \Delta(\bar{B}^0 \to \pi^+K^-) = -\Delta(B_s \to K^+\pi^-),\\
&(6) \quad \Delta(\bar{B}^0 \to \pi^0\bar{K}^0) = -\Delta(B_s \to K^0\pi^0)\\
&\qquad = 3\Delta(\bar{B}^0 \to \eta_8\bar{K}^0) = -3\Delta(B_s \to K^0\eta_8). \qquad (16)
\end{aligned}$$

Note that in the SU(3) limit, beside the above relations, there are several other relations for the branching ratios; that is, some of the decay amplitudes are actually equal in the SU(3) limit. We have

$$\begin{aligned}
&\Gamma(B_s \to \pi^+\pi^-) = 2\Gamma(B_s \to \pi^0\pi^0),\\
&\Gamma(\bar{B}^0 \to \pi^0\bar{K}^0) = 3\Gamma(\bar{B}^0 \to \eta_8\bar{K}^0),\\
&\Gamma(B_s \to K^0\pi^0) = 3\Gamma(B_s \to K^0\eta_8). \qquad (17)
\end{aligned}$$

The last two equalities for the decay rate involve η_8 which mixes with η_1. It will be difficult to carry out these tests. The branching ratio for the first one may be small due to pure annihilation contributions, although it has to be tested independently. This test will also be difficult to carry out.

If it turns out that the annihilation contributions are all small as can be tested in $B^- \to K^-K^0$, $B_s \to \pi^+\pi^-$ and $B_s \to \pi^0\pi^0$, there are additional relations for the rate differences. We find

$$\begin{aligned}
&(1) \approx (4),\\
&(2) \approx (5),\\
&(6) \approx \Delta(\bar{B}^0 \to \pi^0\pi^0) \qquad (18)
\end{aligned}$$

In the limit that annihilation contributions are small, it is difficult to perform tests related to (1), (3) and (4) because the decay rates involved are all small. The equalities of (2) and (5) provide the best chances to test the SM.

The above non-trivial equalities do not depend on the numerical values of the final-state rescattering phases. Of course, these relations are true only for the SM with three generations. Therefore, they provide tests for the three generation model.

The relations obtained above hold in the SU(3) limit. Let us now study how these relations are modified when SU(3) breaking effects are included. Since no reliable calculational tool exists, in the following we will use a factorization approximation neglecting the annihilation contributions to estimate the SU(3) breaking effects for (2) for illustration. We have [20]

$$\begin{aligned}
T^{B_d}_{\pi^-\pi^+}(d) &= \mathrm{i}\frac{G_F}{\sqrt{2}} f_\pi F_0^{B\pi}(m_\pi^2)(m_B^2 - m_\pi^2)\\
&\quad\times\left[\frac{1}{N}c_1 + c_2 + \frac{1}{N}c_3^{uc} + c_4^{uc} + \frac{1}{N}c_9^{uc} + c_{10}^{uc}\right.\\
&\quad+\frac{2m_\pi^2}{(m_b - m_u)(m_u + m_d)}\\
&\quad\left.\times\left(\frac{1}{N}c_5^{uc} + c_6^{uc} + \frac{1}{N}c_7^{uc} + c_8^{uc}\right)\right],\\
T^{B_s}_{K^+K^-}(s) &= \mathrm{i}\frac{G_F}{\sqrt{2}} f_K F_0^{BK}(m_K^2)(m_B^2 - m_K^2)\\
&\quad\times\left[\frac{1}{N}c_1 + c_2 + \frac{1}{N}c_3^{uc} + c_4^{uc} + \frac{1}{N}c_9^{uc} + c_{10}^{cu}\right.\\
&\quad+\frac{2m_K^2}{(m_b - m_u)(m_u + m_s)}\\
&\quad\left.\times\left(\frac{1}{N}c_5^{uc} + c_6^{uc} + \frac{1}{N}c_7^{uc} + c_8^{uc}\right)\right]. \qquad (19)
\end{aligned}$$

The amplitudes $P(d,s)$ are obtained by setting $c_{1,2} = 0$ and replacing c_i^{uc} by c_i^{tc}.

Using the fact $m_\pi^2/(m_u + m_d) \approx m_K^2/(m_u + m_s)$, we obtain

$$\begin{aligned}
&\Delta(\bar{B}^0 \to \pi^+\pi^-)\\
&\approx -\frac{(f_\pi F_0^{B\pi}(m_\pi^2))^2}{(f_K F_0^{B_sK}(m_K^2))^2}\frac{\lambda_{\pi\pi}}{\lambda_{KK}}\Delta(B_s \to K^+K^-), \qquad (20)
\end{aligned}$$

In the above, the final-state interaction phases for the different amplitudes have been assumed to be zero. We point out that as long as these phases satisfy SU(3) symmetry relations, the above equation does not change.

Similarly we also have

$$\begin{aligned}
&\Delta(\bar{B}^0 \to \pi^+\pi^-)\\
&\approx -\frac{(f_\pi F_0^{B\pi}(m_\pi^2))^2}{(f_K F_0^{B\pi}(m_\pi^2))^2}\frac{\lambda_{\pi\pi}}{\lambda_{\pi K}}\Delta(\bar{B}^0 \to \pi^+K^-)\\
&\approx \frac{(f_\pi F_0^{B\pi}(m_\pi^2))^2}{(f_\pi F_0^{B_sK}(m_\pi^2))^2}\frac{\lambda_{\pi\pi}}{\lambda_{\pi K}}\Delta(\bar{B}_s \to K^+\pi^-). \qquad (21)
\end{aligned}$$

The form factors are usually assumed to have the pole-form dependence on q^2. For the above cases the form factors are approximately equal to their values at $q^2 = 0$ because the B meson mass is much larger than the π and K meson masses. For the same reason, $\lambda_{\pi\pi}/\lambda_{\pi K} \approx 1$. Independent of the specific value for the ratio $r = F_0^{B\pi}(0)/F_0^{B_sK}(0)$, we obtain the following relations:

$$\begin{aligned}
&\Delta(\bar{B}^0 \to \pi^+\pi^-) \approx -\frac{f_\pi^2}{f_K^2}\Delta(\bar{B}^0 \to \pi^+K^-),\\
&\Delta(B_s \to K^+K^-) \approx -\frac{f_K^2}{f_\pi^2}\Delta(B_s \to \pi^-K^+). \qquad (22)
\end{aligned}$$

The first equality above has already been obtained before [20]. The ratio r is expected to be about one. If this is indeed the case, one would obtain $\Delta(\bar{B}^0 \to \pi^+\pi^-) \approx \Delta(B_s \to K^+\pi^-)$.

It has been shown that the normalized asymmetry, that is, the rate difference divided by the averaged particle and anti-particle branching for $\bar{B}^0 \to \pi^+K^-$, can be as large as 20% [3,23]. Such a large value can be measured in the future at B factories. The standard model can be tested using the relations discussed in this section.

5 Conclusions and discussions

Several methods proposed to measure the fundamental parameter γ in the KM unitarity triangle depend on the assumption that $A(B^- \to \pi^-\bar{K}^0) = \bar{A}(B^+ \to K^0\pi^+)$. In order for this assumption to hold it is not sufficient to only require the annihilation contributions to be small. One also has to show that the tree amplitude only receives annihilation contributions. In this paper we have shown that these two conditions can be separately tested at B factories in the near future. Of course, one should also keep an open mind for the possibility that the annihilation contribution $A_{\overline{15}}$ is not small, but the total tree contribution $C_3^{\mathrm{T}} - C_6^{\mathrm{T}} + 3A_{\overline{15}}^{\mathrm{T}} + C_{\overline{15}}^{\mathrm{T}}$ yet is small. This can also be tested by measuring the $B^- \to K^- K^0$ branching ratio because the dominant contribution to the amplitude is proportional to the tree amplitude for $B^- \to \pi^- \bar{K}^0$.

We have also derived several useful relations using SU(3) symmetry without any additional dynamic-model assumptions about the amplitudes. These relations will provide further tests for the standard model of CP violation. The SU(3) symmetry is expected to be broken in reality. Therefore the validity of some of the methods for measuring γ and the relations derived in this paper remain to be studied.

Let us conclude with a discussion of the validity of SU(3) relations for B meson decays. We have used a factorization approximation to give some idea of how the SU(3) breaking effects affect the results. We stress that these results are only indicative. One should not exclude the possibility that the experimental results obtained will be actually closer to the SU(3) limit results. Even though we know that SU(3) symmetry is broken in reality, the breaking pattern may be much more subtle than a simple decay constant rescaling as indicated by our factorization calculations in the previous sections. To see why this might happen let us consider the $B^- \to D^0\pi^-$ and $B^- \to D^0 K^-$ decays.

We find that in the SU(3) limit the ratio $R = Br(B^- \to D^0 K^-)/Br(B^- \to D^0\pi^-)$ is equal to $|V_{us}/V_{ud}|^2(\lambda_{DK}/\lambda_{D\pi})$. The value $R = 0.049$ obtained in the SU(3) limit is more closer to the experimental central value of $0.055 \pm 0.015 \pm 0.005$ from CLEO [26] than the factorization estimate with SU(3) breaking, $R \approx (f_K^2/f_\pi^2)|V_{us}/V_{ud}|^2(\lambda_{DK}/\lambda_{D\pi}) \approx 0.07$. Of course, the experimental result is consistent with both predictions at present. The point of this example is that one should be careful with a factorization estimate of the SU(3) breaking effects. SU(3) relations may turn out to be better than expected. We have to wait for more experimental data to provide us with more information.

The above discussion also applies to the relation between the tree amplitude A^{T} for $B^- \to \pi^-\pi^0$ and the $I = 3/2$ tree amplitude $A_{3/2}^{\mathrm{T}}$ for the $B^- \to \pi^0 K^-$ and $B^- \to \pi^- \bar{K}^0$ decays. The experimental value may turn out to be closer to the SU(3) limit result than the relation estimated by factorization [4,12], $A_{3/2}^{\mathrm{T}} = (f_K^2/f_\pi^2)|V_{us}/V_{ud}|^2 A^{\mathrm{T}}$. This also has important implications for the determination of γ. Any method to determine γ using this relation should be analyzed with care.

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